Celestial Mechanics During the Two Decades 1957 – 1977

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Introduction

NEWTON'S discovery of the law of gravitation and the laws of motion created the concept of dynamical systems and the field known today as celestial mechanics. (Astrodynamics, orbital mechanics, and space dynamics are recently introduced expressions, emphasizing the engineering aspects of celestial mechanics. This review will attempt to include "pure" and "applied" celestial mechanics.)

Copernicus, Brahe, Kepler, Galileo, Newton, Euler, Laplace, Lagrange, Hamilton, Jacobi, Poincaré, Hill, Birkhoff, Whittaker, Moulton, Brouwer, Clemence, Eckert, Herrick, Stumpff, and Chebotarev are some of the giants on whose shoulders we perch. This admittedly imcomplete list and its order are important. The first fourteen offered the background. The work of the last seven is included in the review, since they contributed directly to celestial mechanics in the past twenty years. This review will concentrate on the results and problems in the past twenty years rather than on the names of the individual contributors, excepting the authors of major books.

The preceding list also may aid the discussion of a criticism uttered so frequently by the ignorami: "Celestial mechanics is a dead field since Newton 'solved all problems'." Has Kepler been asked why he is working on celestial mechanics when Copernicus "solved all problems"? And if Newton solved all problems, what have the aforenamed giants and *we* done in the past 300 years? But let us move to Whittaker, who certainly "solved all problems," in which case Moulton, Brouwer, Clemence, and Herrick have done nothing. If so, how may one report on advances in this field in the past 20 years? The answer is, of course, well known to the cognoscenti: celestial mechanics is alive and well and living at several centers. Significant contributions are being made and new problems are being solved daily, using techniques Newton never even dreamed about. Furthermore, new and challenging unsolved problems emerge continuously, assuring that celestial mechanics will continue to be an exciting field of research.

What Is New in Celestial Mechanics?

For the last twenty years, beginning with the launching of the first artificial satellites, we experienced the following important innovations:

1) Electronic computers have increased in speed, storage capacity, and versatility.

2) We have gained high accuracy in measuring distances directly with radar and laser technology.

3) Celestial mechanics became an experimental science, designing its own experiments, finding new results, proving theories, and increasing the accuracy of astronomical constants.

4) New analytical techniques were discovered and became operational, including averaging methods, regularization, Lie series, KAM theory, canonical operations in the extended phase space, resonance theory, perturbations in rectangular coordinates, asymptotic expansions and the theory of singular perturbations, search for new integrals, and new concepts of local and global stability, to mention just a few.

5) Close approach trajectories became of considerable operational importance, requiring the introduction of new analytical and computational methods.

6) The restricted many-body problem re-emerged as the fundamental dynamical system of considerable practical physical importance.

7) Direct and inverse high-order satellite theories were developed for satellite orbit computations and for determination of the geopotential.

8) Sophisticated statistical methods evolved to handle continuous or very large numbers of discrete data points, using filtering techniques and allowing real-time orbit determination.

9) To accommodate the increased interest in celestial mechanics, several research centers were established, working groups were formed, journals were started, regular meetings were held, and institutes were conducted. The increase in the number of workers in the field may be estimated to be sixfold during the past twenty years.

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This list is not complete, and it is not arranged according to importance. *De gustibus non est disputandum,* and the reader's list may contain different items; after all, one man's analytical existence proof may be another man's emperor's clothes, and one man's high-order, sophisticated numerical integration may be another man's meaningless, divergent approximation. Some additional details concerning the preceding items follow:

1) Both general and special perturbation techniques benefited from computer development. Algebraic manipulations in general, computerized Poisson and Fourier series, and compression and storage of ephemerides by Chebyshev polynomials might be mentioned as the most significant nonnumerical uses of computers assisting the development of general perturbation theories (lunar and planetary theories). High, variable-order, and variable step-size Runge-Kutta, recurrent power series, and other numerical integration methods were developed, allowing long-time integration of the solar system (10^6-10^7 years) and attempting to show its Laplacian stability. The semianalytical dynamical theories certainly could not have gained their just popularity without advances in computers; neither could the librational motions of Neptune and Pluto be discovered, nor their minimum distances established as 18 a. u. Leibnitz maintained that "it was unworthy of excellent men to lose hours like slaves in the labor of calculations which could be relegated to anyone else if machines were (available and) used." Indeed, his desk calculator of 1693 was considered a significant progress, but could it be compared to the progress in electronic computers during the last twenty five years?

2, 3) We have been accustomed to gaining about one significant figure per century for basic astronomical constants since the seventeenth century, but in the last twenty years we gained two to four significant figures. Reanalysis of old and use of new observations increased the length of validity and accuracy of planetary and satellite ephemerides, improved our knowledge of the astronomical unit, offered tests of relativity theory, allowed checks on the variation of the constant of gravity, and increased the accuracy of our knowledge of planetary masses. It also gave new information on the rotation of planets, on the motion of the Earth's spin axis, and on other physical parameters. Progress in rigid-body dynamics also should be mentioned: high-degree-of-freedom systems were analyzed successfully with variable inertia tensors.

4) This item contains many advances, several of which were listed previously. One outstanding accomplishment was the introduction of Lie series in perturbation theory, a systematic, general, and explicit method. Another was the KAM result concerning quasiperiodic motions offering a new approach to stability research. More abstract and sophisticated, but nevertheless important, results were obtained regarding collinear three- and four-body motions, three-body motions with zero total energy, and the asymptotic behavior of many-body gravitational systems. Several new local isolating integrals were found (without violating Bruns' and Poincaré's theorems of nonintegrability). Matched asymptotic methods were applied to near triple collisions, a nonregularizable problem of fundamental importance, creating major difficulties in the analytical and numerical treatments of many-body gravitational problems. The introduction of the method of general perturbations using rectangular coordinates eliminated singularities occuring with orbital elements.

Significant progress was made in the discovery and classification of periodic solutions of dynamical systems occuring in celestial mechanics. E. Strömgren's families in the restricted problem of three bodies were completed, many new families were discovered, and their (linear) stability parameters were found. The success of continuation of these families into the general problem of three bodies has shown that, when Birkhoff considered the restricted problem as one of the fundamental, "generating" problems of celestial mechanics, he did not exaggerate its importance.

5) Brouwer in 1962 announced three fundamental unsolved problems in celestial mechanics: the problem of small divisors, the computation of trajectories connecting the vicinity of two bodies, and the explanation of the Kirkwood gaps in the asteroid belt. The first, an old problem in celestial mechanics (the problem of small divisors), will be discussed in item 7. The second problem is characteristic of space research, where such trajectories are often mission requirements, such as Earth-to-Moon trajectories, interplanetary fly-by and swing-by orbits, etc. Because of the Newtonian law of gravitation, the forces acting on a space probe are inversely proportional to the square of the distances. Consequently, close approaches result in close singularities and very large forces. Such two-body singularities, however, may be eliminated by the technique of regularization, which has not been used until recently. (Note that using Encke's method, with rectifying at every step, accomplishes the same, and universal variables may be considered as a special case of regularization.) Regularization (transforming the dependent variable and often the independent ones, also) eliminates the singularities as well as linearizes the two-body problem. Perturbed two-body problems appear as perturbed harmonic oscillators with analytical techniques available for their solutions. The introduction and discovery of various regularization techniques resulted also in an unexpected (but today, of course, completely understood) bonus: greatly increased accuracy and increased stability in numerical integration. In fact, even when close approaches are not expected, the use of regularized and universal variables is recommended to increase the accuracy and reduce the time of numerical integrations. Application of these variables in boundary-value problems, in optimization, in the solution of Lambert's problem, in relative motion computation, in satellite theory, and in other fields was found to be beneficial.

Asymptotic expansions, theory of singular perturbations, matching, and the use of multiple scales borrowed from fluid mechanics entered celestial mechanics with remarkable success. The close approach and the small divisor problems mentioned previously are amenable to these techniques, which also were used to establish new periodic orbits in the restricted problem of three bodies.

6) Birkoff's emphasis on the restricted three-body problem was quoted previously. Since the mass of space probes is always orders of magnitude smaller than that of the other masses participating in space trajectory problems, the assumptions of the restricted problem are more applicable in space research than in classical celestial mechanics. This means that the dynamical system may be uncoupled, and the three- (or six-) degrees-of-freedom motion of the probe may be computed (assuming that the orbits of all of the other participating bodies are known).

Modifications of the classical circular restricted problem of three bodies to the elliptic motion of the primaries, to primaries with variable masses, and to primaries devoid of spherically symmetric mass distributions (oblate spheroids) received considerable attention. The role of the equilibrium points and their stability was studied in connection with the establishment of stable periodic orbits around these points and regarding minimum fuel requirements for stationkeeping. The existence of families of horseshoe-type orbits was established numerically and analytically, not only for the restricted problem of three bodies but also for the Sun-Jupiter-asteroid problem. The classical termination principle was re-examined and modified, and new terminations and bifurcations (sometimes via regularization) were established. New periodic orbits of considerable interest for space missions were found, such as figure-eight orbits, orbits encircling first the Earth and then the Moon several times, and large-amplitude, three-dimensional halo orbits around the collinear equilibrium points.

Families of two- and three-dimensional asymmetric periodic orbits were discovered in the restricted problem, existence proofs were given of second- and third-kind periodic orbits for arbitrary values of the mass parameter, conditions of nonlinear stability of the triangular equilibrium points were established, the role and possible utilization of the Jacobian integral in the elliptic restricted problem were investigated, and a detailed study of the families of periodic orbits for the Sun-Jupiter and for the Earth-Moon problems has been completed.

7) Brouwer's fundamental paper on the motion of artificial satellites was published in 1959. This may be considered the first completely analytic solution of the problem, having closed form in the inclination and in the eccentricity. It soon was modified by inclusion of second-order, short-period terms in the semimajor axis, by using modified Delauney and Poincaré elements, by inclusion of drag and radiation effects, by several higher order corrections, and by use of new variables eliminating singularities. An interesting feature of the solution is known as the critical angle of inclination; when the angle of inclination is less than 63.435 deg, the motion of the argument of the perigee is in the positive direction, and when it is higher, the perigee moves in the negative direction. This fact lead to considerable controversy, since some numerical experiments could not verify this finding.

Hill's idea of starting a perturbation theory with a better intermediate orbit than a Keplerian ellipse was quickly applied to artificial satellite theories with some success, depending on the possibility of obtaining higher order theories. Satellite theories applicable to lunar satellites present serious problems because of large perturbations by the Sun and the Earth and because of the highly irregular gravitational potential of the Moon. In spite of these difficulties, satisfactory theories now exist for artificial satellites of the Moon.

The inverse problem is to determine the gravitational potential of a body from satellite orbit observations. This problem is closely related to items 2, 3, and 8, and it may be considered one of the great scientific victories of the past twenty years. The conventional approach is to represent the gravitational potential by harmonics and to determine the coefficients of the expansion (together with other unknowns of the problem) from the observational results. It was established that low- and high-order harmonics (say, orders of 20 or 30) can be important and may be detected in this way for the geopotential (in contradiction to the fundamental hypothesis of geodesy). Tesseral harmonics and the motion and stability properties of synchronous (24-h) satellites also were established. From lunar satellite observations, large mass concentrations (mascons) were found under the maria, once again using the inverse technique. Unified theories of spin-orbit coupling and orbit-orbit resonances were offered, and Mercury's 3:2 spin-orbit resonance was established.

8) Techniques of orbit determination and data procession were completely revolutionized. Using radar and laser techniques (see item 3), celestial mechanics did not rely any more on the measurements of angles only. In this way, not only new methods of data handling but also new analytical methods had to be introduced to utilize simultaneous distance and angular measurements. Determination of observational station coordinates with very high accuracy became a reality, and navigational uses of satellites became a standard operation. Tectonic plate activity and surface motion of the oceans can be determined today, also with high accuracy.

9) The review of the exciting progress during the past twenty years is not complete without the names of the contributors. A cursory counting, however, yielded approximately 120 names of presently active research workers in celestial mechanics, inclusion of which, unfortunately, is not possible here. On the other hand, an estimate of twenty active contributors in 1950 probably would be too high. About fifty of the major books published in celestial mechanics during the past twenty years are cited at the end of this article. It is difficult to estimate the number of major books on celestial mechanics from Copernicus to 1957, but it may be the same number or less than appeared during the last twenty years.

Future Expectations

1) Historically, in celestial mechanics, the calculations could be performed with more accuracy than the observations. This is no longer true. If the motions of the Moon and of artificial Earth satellites are measured within a few centimeters' accuracy, then our computational methods certainly have fallen behind. This trend eventually will be reversed, and analytical and numerical techniques will be developed to increase computational accuracies above the accuracy of observations. More accurate planetary and lunar theories are needed with long-time validity, and the range of validity of numerical integrations should be increased.

2) Independently of measurement accuracies, the problem of long-time prediction must be faced. This is a fundamental problem of non-integrable dynamical systems (such as encountered in celestial mechanics). Realizing the importance of selecting the proper model and its influence, we must have results of long-time validity regarding the behavior of our system. Numerical techniques do not seem to be able, in principle, to answer questions concerning the motion for arbitrary long-time because of error accumulations. The nonintegrable dynamical systems of celestial mechanics do not possess "solutions" that can be expressed with the presently known functions of analysis. Consequently, even if our physical models were perfect, the long-time behavior of our system in general could not be established quantitatively today. Qualitative methods are available and may hold some hope for answering questions on stability, on permissible ranges of motion, on capture of natural satellites, etc.

3) Our knowledge regarding the astronomical constants is still meager. Therefore, our ability to build the proper models is limited. Consider, for instance, the effect of drag on artificial satellites. Since the drag depends on the atmospheric density, which, in turn, is affected by the activities of the Sun, a drag model with long-time validity cannot be built today. Another example is the determination of higher-order gravitational harmonics. Here, one might be faced with another problem that is insoluble with our present-day techniques. The actual problem is to determine the gravitational potential and not the possibly divergent coefficients of its assumed form of harmonic expansion. This problem has no complete solution, since it is known that, from an orbit, the field may not be determined uniquely. Nevertheless, other forms of the potential function may be found which offer better descriptions of the physical situation.

4) Considerable improvements may be expected regarding the evaluation of planetary influences on the Moon's motion and in the computation of its librations. Similarly, increased accuracies are expected in the theories of the motions of other natural satellites of the solar system.

5) Computer-controlled telescopes and computerized data handling in astronomy are expected to furnish additional data on the solar system. This, in conjunction with radar and laser observations, certainly will allow the additional study of relativity effects. Analytical progress in relativistic celestial mechanics is one of the challenges toward which workers in the field look with great interest and expectation.

6) Further improvements in electronic computers should allow an extension of the range of meaningful and accurate numerical integration. Long-term numerical stability will be increased by the use of proper variables. Integration in the extended phase space with zero Hamiltonian will increase accuracy. It is expected that new integration methods will emerge, along with methods to control global error propagation.

Epilog

The renaissance of celestial mechanics attracted high-caliber research workers from a variety of fields. Some of the visitors made significant contributions, fertilizing the ancient ground of celestial mechanics with new methods. Is further progress of an asymptotic nature? It is known that astute research workers are attracted to fields where new results can be obtained with relative ease. In Einstein's words, "scientists are unscrupulous opportunists." Is there hope for great discoveries in celestial mechanics? Or can only small and unimportant "epsilon" improvements be expected at the price of very large investments of labor? It depends on the availability of new analytical and numerical techniques. Celestial mechanics was responsible, during the history of natural science, for bringing forth new mathematical methods several times. (A striking example is Gauss' introduction of the method of least squares in order to find a lost asteroid.) This may happen again. Great new discoveries in celestial mechanics will need new techniques and new approaches. Possible new avenues may be opened up by the extensive use of techniques of statistical mechanics, of utilization of qualitative methods, and by broad-based numerical experimentation.

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